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LETTER TO THE EDITOR

Ground state of a spin-$\frac{1}{2}$ neutral particle with an anomalous magnetic moment in a two-dimensional electrostatic field

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Abstract. It is shown that a relativistic spin-$\frac{1}{2}$ neutral particle with an anomalous magnetic moment moving in an x–y plane in a two-dimensional electrostatic field: $\boldsymbol{E}=(E_x(x,y), E_y(x,y), 0)$ has $(N-1)$-fold degeneracy of the ground state. The degree of degeneracy of the ground state is defined by the value of the anomalous magnetic moment and a linear charge density of the filament creating this electrostatic field.

Aharonov and Casher [1] have shown that, in the non-relativistic limit, the wavefunction of a neutral particle with an anomalous magnetic moment moving in a closed path which encircles an infinitely long filament carrying a uniform charge density, develops the phaseshift. Recently this phenomenon has been observed experimentally [2]. The similarities and differences between Aharonov–Casher and Aharonov–Bohm [3] effects are studied in [4–6] within the framework of classical mechanics and in [7, 8] within the framework of quantum mechanics. In particular, there were noted some similarities between the Aharonov–Bohm effect in a two-dimensional magnetic field $\boldsymbol{B}=(0, 0, B_z(x,y))$ and the Aharonov–Casher effect in a two-dimensional electrostatic field $\boldsymbol{E}=(E_x(x,y), E_y(x,y), 0)$. Earlier, it was shown by Aharonov and Casher [9] that both the relativistic and non-relativistic spin-$\frac{1}{2}$ charged particle moving in an x–y plane under the influence of a perpendicular magnetic field $\boldsymbol{B}=(0, 0, B_z(x,y))$ has $(N-1)$-fold degeneracy of the ground state. The degree of degeneracy of the ground state is defined by the total magnetic flux in units of quantum flux. Therefore, there appears to be some interest in considering the ground state of a spin-$\frac{1}{2}$ neutral particle with an anomalous magnetic moment moving in an x–y plane in a two-dimensional electrostatic field $\boldsymbol{E}=(E_x(x,y), E_y(x,y), 0)$. Here, we consider the relativistic case only, but the results hold true also in a non-relativistic case. The magnetic moment for a neutral particle is the anomalous one, so the Dirac equation for this particle includes interaction with the electromagnetic field only in Pauli's form [10] and is as follows:

$$(\gamma^\mu p_\mu - m)\phi = \frac{\mathrm{i}}{2}\mu\sigma^{\mu\nu}F_{\mu\nu}\phi \tag{1}$$

where $c=\hbar=1$, μ is an anomalous magnetic moment, $\sigma^{\mu\nu}=\frac{1}{2}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)$ and the Dirac matrices γ^μ are chosen in a standard representation. Considering stationary solutions of equation (1), we have

$$E\phi = H_D\phi \equiv \begin{vmatrix} m & \boldsymbol{\sigma}\cdot\boldsymbol{A}^+ \\ \boldsymbol{\sigma}\cdot\boldsymbol{A}^- & -m \end{vmatrix} \begin{vmatrix} \phi_1 \\ \phi_2 \end{vmatrix} \tag{2}$$

where $\boldsymbol{A}^\pm = \boldsymbol{p} \mp \mathrm{i}\boldsymbol{E}$.

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Consequently, the large ϕ_1 and small ϕ_2 components of the Dirac spinor ϕ satisfy the factorized equations

$$(E^2-m^2)\begin{vmatrix}\phi_1\\ \phi_2\end{vmatrix}=\begin{vmatrix}(\boldsymbol{\sigma}\cdot\boldsymbol{A}^+)(\boldsymbol{\sigma}\cdot\boldsymbol{A}^-) & 0\\ 0 & (\boldsymbol{\sigma}\cdot\boldsymbol{A}^-)(\boldsymbol{\sigma}\cdot\boldsymbol{A}^+)\end{vmatrix}\begin{vmatrix}\phi_1\\ \phi_2\end{vmatrix}\tag{3}$$

that is the large ϕ_1 and small ϕ_2 components of the Dirac spinor ϕ are supersymmetric partners [11, 12]. Equations (3) have identical energy spectra except for the ground state. When supersymmetry is unbroken, the ground state wavefunction with the ground state energy eigenvalue $E^2=m^2$ is defined by one of the following:

$$(\boldsymbol{\sigma}\cdot\boldsymbol{A}^-)\phi_1=0\tag{4a}$$

$$(\boldsymbol{\sigma}\cdot\boldsymbol{A}^+)\phi_2=0.\tag{4b}$$

Since the electrostatic field has the form: $\boldsymbol{E}=(E_x(x,y), E_y(x,y), 0)$ and we consider the motion of a neutral particle in the x-y plane ($p_z=0$), (4a) and (4b) assume the following forms:

$$[(p_x+\mathrm{i}\mu E_x)+\mathrm{i}\sigma_z(p_y+\mathrm{i}\mu E_y)]\phi_1=0\tag{5a}$$

$$[(p_x-\mathrm{i}\mu E_x)+\mathrm{i}\sigma_z(p_y-\mathrm{i}\mu E_y)]\phi_2=0.\tag{5b}$$

The field strength is given by $E_x(x,y)=\partial_x\varphi(x,y)$, $E_y(x,y)=-\partial_y\varphi(x,y)$ and $E_z=0$, where the electrostatic potential satisfies the Laplace equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\varphi=-4\pi\sigma(x,y)\tag{6}$$

where $\sigma(x,y)$ is the charge density per unit volume. We suppose that the charge is localized in some region on the x-y plane and is infinity in the z direction.

After the substitutions

$$\phi_1=\mathrm{e}^{\mu\varphi}f=\exp\left[\frac{\mu}{2\pi}\int\sigma(\xi,\eta)\ln\left(\frac{[(x-\xi)^2+(y-\eta)^2]^{1/2}}{r_0}\right)\mathrm{d}\xi\,\mathrm{d}\eta\right]f\tag{7a}$$

$$\phi_2=\mathrm{e}^{-\mu\varphi}g=\exp\left[-\frac{\mu}{2\pi}\int\sigma(\xi,\eta)\ln\left(\frac{[(x-\xi)^2+(y-\eta)^2]^{1/2}}{r_0}\right)\mathrm{d}\xi\,\mathrm{d}\eta\right]g.\tag{7b}$$

Equations (5a) and (5b) lead to

$$(\partial_x+\mathrm{i}\sigma_z\partial_y)f(x,y)=0\tag{8a}$$

$$(\partial_x+\mathrm{i}\sigma_z\partial_y)g(x,y)=0.\tag{8b}$$

Thus f and g (or more exactly, the components of f and g) are the entire functions of $x+\mathrm{i}y$ and $x-\mathrm{i}y$. To take into account the asymptotic behaviour of the exponents in (7a) and (7b) the conditions of the square-integrability of f and g have the form

$$\lim_{r\to\infty} r^2|f|^2\left(\frac{r_0}{r}\right)^{2\mu\lambda/2\pi}=0\tag{9a}$$

$$\lim_{r\to\infty} r^2|g|^2\left(\frac{r_0}{r}\right)^{-2\mu\lambda/2\pi}=0\tag{9b}$$

where λ is the linear charge density: $\lambda=\int\sigma(\xi,\eta)\,\mathrm{d}\xi\,\mathrm{d}\eta=2\pi/\mu(N+\varepsilon)$, N is a positive integer, $0<\varepsilon<1$. Hence, f or g must be a polynomial whose degree is no larger than $N-1$. Moreover, ϕ_1 and ϕ_2 are the components of the Dirac spinor ϕ, which are connected due to system (2). Consequently, apart from the normalizablity of f (or g) the other function g (or f) should vanish. Therefore, in the case $\mu\lambda > 0$ we find that ϕ_2 is a normalizable solution and from (2) we conclude that $\phi_1 = 0$ only at $E = -m$. Analogously, in the case $\mu\lambda < 0$, ϕ_1 is a normalizable solution and $\phi_2 = 0$ only at $E = m$.

Thus we have shown that a relativistic spin-$\frac{1}{2}$ neutral particle with an anomalous magnetic moment moving in the x–y plane in a two-dimensional electrostatic field: $\boldsymbol{E} = (E_x(x, y), E_y(x, y), 0)$ has $(N-1)$-fold degeneracy of the ground state. The degree of degeneracy of the ground state is defined by the value of the anomalous magnetic moment and a linear charge density of the filament creating this electrostatic field. Note, that in the three-dimensional central electrostatic field there is infinite degeneracy of the ground state [13].

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